Mathematics textbooks – the link between the intended and the implemented curriculum?

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Textbooks are a predominant source in mathematics classrooms in Sweden as well as in many other countries. Consequently, they often determine what school mathematics is and also what mathematics is for students and teachers. They can also have a prominent position in reform of mathematics curriculum since the development of textbooks and other curriculum materials can be seen as a quick and easy way to change teaching. This paper reports from a study of textbooks as a possible link between educational goals and classroom activities – the potentially implemented curriculum. The aim is to contribute to the discussion about the role of textbooks in mathematics education.

Introduction

Textbooks are a most important feature of the teaching of mathematics because of their close relation to classroom instruction. The textbooks identify the topics and order them in a way students should explore them. They also attempt to specify how classroom lessons can be structured with suitable exercises and activities. Hence, textbooks are designed for the purpose to help teachers to organize their teaching.

There is a good deal of evidence that many teachers like the security and freedom from responsibility that a text series provides. […] when using a text series, teachers need not involve themselves in ordering the topics, in ensuring that notation is consistent, nor in concerning themselves whether a student will have met the necessary pre-requisites for a new topic (Love & Pimm, 1996, p. 384). Some mathematics textbooks contain only problems and exercises. These kind of books require support from a teacher who will play a central role in mediating the text to the students (Love & Pimm, 1996). There are also textbooks that have a mix of theoretical notes, problems, exercises and other assignment. Such a book “seems to be a teacher in itself” (van Dormolen, 1986, p. 141). But is it possible to write a teacher-proof text? A more global question is if textbooks, themselves, can contribute to mathematics learning. The issue is especially relevant to Sweden where students and teachers seem to be very dependent on textbooks. Content as well as preparation and organisation of the lesson is very much dictated by textbooks. They define ‘school mathematics’ as well as the ‘learning path’ for the majority of students, at least in lower and upper secondary school (Skolverket, 2003). The situation in Sweden is however not unique. Previous research on textbooks and teachers’ use of textbooks shows, among other things, that:

(a) Mathematical topics in textbooks are most likely presented by the teachers (Freeman & Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman, 2003);
(b) Mathematical topics not included in textbooks are most likely not presented by the teachers (Freeman & Porter, 1989; Reys et al., 2003);
(c) Teachers’ pedagogical strategies are often influenced by the instructional approach of the material (Reys et al., 2003);
(d) Teachers’ sequence of instruction are often parallel to that of the textbook (Freeman & Porter, 1989).
(e) Teachers report that textbooks are a primary information source in deciding how to present content (Schmidt et al., 2001)
With these results as a background, I believe that an increased awareness of textbooks and how they are used is crucial for understanding the process of teaching and learning mathematics. If one considers a reform of the mathematics curriculum it is therefore important to understand the role of textbooks. In this paper, I will briefly present a study of textbooks that I conducted in 2003. The development of a textbooks series, a commonly used schoolbook in Sweden, is portrayed in the light of the curriculum development (Johansson, 2003).

**The curriculum model**

In part, textbooks provide indications of students’ opportunities to learn. The study of textbooks was therefore important in the research design of the *Third International Mathematics and Science Study*, TIMSS. In the curriculum model, textbooks are regarded as the potentially implemented curriculum, the link between aims and reality (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

Figure 1: Textbooks and the tripartite model (Valverde et al., 2002, p.13)

In this model (figure 1), the intended curriculum is at the educational system level. It is seen in national policies and official documents which reflect societal visions, educational planning, and official or political sanctioning for educational objectives. Intention and objectives at the level of the teacher and the classroom activity are considered as the implemented curriculum. The potentially implemented curriculum, which is represented by textbooks and other organized resource material, is regarded as a link between these two levels (Robitaille et al., 1993; Schmidt et al., 1997).

The conceptual framework for the TIMSS Curriculum Study is based on the view of the textbooks as mediators between general intentions and classroom instruction. But what is the relationship between textbooks and the intended curriculum? Are textbooks, in general, appropriate tools for translating guidelines that are stated by educational authorities into activities in classrooms?
A case study of the development of a Swedish textbook series

In Sweden, the objectives of teaching and learning mathematics in compulsory school are expressed and explicitly stated by the National Agency of Education in a national curriculum (the Swedish term is läroplan). During the last thirty years, the curriculum has been revised two times, 1980 and 1994. For the purpose to examine the link between the intended curriculum and textbooks, I made a content analysis of a textbook series. The development of the textbook series, a commonly used schoolbook in Sweden, was evaluated in light of the curriculum development. The aim was to examine to what extent a reform of the curriculum influences the development of mathematics textbooks. The study is published in full in the licentiate thesis Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum (Johansson, 2003).

Three editions of the textbook series, which have been on the market since the beginning of the 70s’, are chosen. The editions that are published in 1979 and 1985 consist of two books each, one for the general course and one for the more advanced course. The third edition from 2001 consists of one book. There are two reasons why I chose this particular textbook: a) even though almost thirty years passed between the first and latest edition, the group of authors is the same all over time; and b) this was one of the two textbook series selected for the TIMSS curriculum study. The textbooks are intended to cover the topic for a school year (year 7) and are designed in a way that facilitates individual work by the students. The chapters have sets of worked examples, exercises, word problems, and summaries of facts. The books also have sections with review and answers to all exercises. Besides that, the new edition has special units at the end of each chapter with, for instance, suggestions for group work and thematic work.

The three curricula that the textbook editions correspond to are from 1969, 1980 and 1994 respectively. The curriculum from 1994 is also the current one. They are quite different in terms of text and volume. During this period of revisions the text has changed from being very descriptive (in 1969) to very general (in 1994) and the number of pages has decreased from over two hundred to less than thirty. However, they all have a section where the objectives (different for each curriculum) of teaching mathematics are stated. One main difference between the curriculum from 1994 and its predecessors is that it emphasises the role of mathematics in our society as well as the historical development of mathematics. The idea that students should learn about the importance of mathematics is evident in the description of the objectives for mathematics as well as in the assessment criteria (Skolverket, 2001).

In the analysis of the textbook series, I found that there is minor agreement between the objectives of mathematics, explicitly stated in the national curriculum, and the content of the textbooks. For example, in the analysis of the most recent edition of the textbook series, I found that it presents very little information about the role of mathematics in our society and only one short story that could belong to the history of mathematics. In a free translation, the story goes like this:

In the twelfth century before Christ, the Egyptians divided the day and the night into twelve hours each. This implied that the length of an hour varied at different times of the year. The system was abandoned in the fourteenth century after Christ. A couple of hundred years before Christ was born, Greek astronomers introduced the partitioning into 60 minutes and 60 seconds. The number 60 came from the Babylonian numerical system (Undvall et al., 2001, p. 236, my free translation).
When and why is mathematics useful? The textbooks chosen for this analysis have, as many other textbooks, theoretical parts. Some of them try to explain when and/or why a specific mathematical topic is useful. In the analysis of the textbook series, I found eight different topics with such explanations. Examples of these explanations can be found in the table below.

Table 1: The topics in the text blocks:

<table>
<thead>
<tr>
<th>Topic</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough estimate</td>
<td>When you are buying things in a store – a rough estimate is helpful if you want to find out how much the costs are.</td>
</tr>
<tr>
<td>Rounding</td>
<td>Stores utilize rounding. If the total sum is 14.47 you must pay 14.50 because there are only whole and half crowns.</td>
</tr>
<tr>
<td>Time</td>
<td>If you want to know how long a trip will take – then you must know how to compute a difference in time.</td>
</tr>
<tr>
<td>Diagrams</td>
<td>The newspapers and the TV often use diagrams to illustrate facts and connections. Diagrams can also be used to illustrate a trip.</td>
</tr>
<tr>
<td>Statistics</td>
<td>Collected data can be more understandable if you compute the mean and the median.</td>
</tr>
<tr>
<td>History</td>
<td>A story about the historical development concerning mathematics.</td>
</tr>
<tr>
<td>Hand-held calculators</td>
<td>Hand-held calculators are used for solving practical problems in every-day life.</td>
</tr>
<tr>
<td>Equations:</td>
<td>Solving equations is relevant mainly in physics and chemistry.</td>
</tr>
</tbody>
</table>

In all books, it was difficult to find attempts of explanations for when and why one can use a specific mathematical knowledge. The distribution of topics in the textbooks is presented in the table below.

Table 2: The number of text blocks associated to the topics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough estimate</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rounding</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Time</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Diagrams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>History</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Hand-held calculators</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Total:</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Moreover, the analysis of the textbooks indicates that the new edition (from 2001) are rather comparable to the old editions (from 1979 and 1985). Special units with for instance problem solving and thematic work are added to the new edition so the number of pages is higher, but the number of exercises is, if we exclude these units, almost the same. This can imply that students are not working through the whole book and it has to be decided which part of the book they should leave out. This decision can be made by: (a) the teacher; (b) the individual student; (c) the student together with the teacher; or (d) the teachers of a school as a collective group. So even if the new edition of the textbook series investigated in this study is more varied with respect to suggestions for students’ activities, it is easy to ignore the parts of the book dedicated to problem
solving and other enrichments. Teachers could use the new book and teach in the same way as with the old one. Students can basically work with the same type of exercises as the students did in the beginning of the 80’s (Johansson, 2003, p. 84).

Discussion
From the case study, one can clearly see that textbooks do not always and in a close way follow the guidelines of the intended curriculum. This implies that it is important to consider the textbooks when planning for a reform of the mathematics curriculum. But we cannot learn about the role of textbooks in mathematics education without taking their use into account. It is therefore important to gain more knowledge about the use of mathematics textbooks in classrooms. Not only how much textbooks are used in relation to other activities should be analyzed but also how and why they are used. Finally, the main elements in the classroom, the teachers and the students, must have the opportunity to reflect upon the characteristics of textbooks and how they use them.

References
Specifically, they investigated the link between mathematical and conditional reasoning skills (i.e., the ability to reason on the basis of if p then q-type statements). An example of this kind of task is the following. If a card is yellow, then it has a circle on it. As we described above, mathematical reasoning relies both on inhibitory processes, and the ability to process abstract and symbolic content. Mathematics content related to the school mathematics curriculum. These are experiences dealing mainly with the structure, sequence, content, and level of competence required from pupils to successfully learn from the school mathematics curriculum (primary or secondary levels). (1987). The underachieving curriculum: Assessing US school mathematics from an international perspective. Champaign, IL: Stipes Publishing Company. Google Scholar. Tattoo M.T., Bankov K. (2018) The Intended, Implemented, and Achieved Curriculum of Mathematics Teacher Education in the United States. In: Tattoo M., Rodriguez M., Smith W., Reckase M., Bankov K. (eds) Exploring the Mathematical Education of Teachers Using TEDS-M Data. Mathematics is a powerful learning tool. As students identify relationships between mathematical concepts and everyday situations and make connections between mathematics and other subjects, they develop the ability to use mathematics to extend and apply their knowledge in other curriculum areas, including science, music, and language. Principles Underlying the Ontario Mathematics Curriculum. The Grade 9 courses in the Ontario mathematics curriculum build on the knowledge of concepts and the skills that students are expected to have by the end of Grade 8. The strands used are similar to those used in the elementary program, with adjustments made to reflect the more abstract nature of mathematics at the secondary level. Second international math has been added to your Cart. Add to Cart. Buy Now. Have one to sell? Sell on Amazon. Flip to back Flip to front. Listen Playing... Enter your mobile number or email address below and we'll send you a link to download the free Kindle App. Then you can start reading Kindle books on your smartphone, tablet, or computer - no Kindle device required. Apple. Android. Windows Phone. Android. To get the free app, enter your mobile phone number. or. PDF | On Jan 1, 2009, Nicole Rene Rigelman published Eliciting student mathematical discourse: Relationships between the intended and enacted curriculum | Find, read and cite all the research you need on ResearchGate. Furthermore, intended outcomes, as when the teachers reverted to more familiar discourse patterns, may not be enacted [25]. This means that there was a discrepancy in the reality of learning experiences afforded to the students despite deliberate and targeted planning. Leveraging reasoning through discourse: A case study in secondary mathematics classrooms. Article.